TRINITY COLLEGE

Semester One Examination, 2020

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

Section One: Calculator-free

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	51	35
Section Two: Calculator-assumed	13	13	100	96	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section One: Calculator-free **35% (51 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(7 marks)**

Two forces are given by $\mathbf{F_1} = -3\mathbf{i} + 5\mathbf{j}$ N and $\mathbf{F_2} = 2\mathbf{i} - \mathbf{j}$ N.

(a) Determine

(i) $\mathbf{F_1} - \mathbf{F_2}$. (1 mark)

(ii) $5\mathbf{F_1} + 10\mathbf{F_2}$. (2 marks)

(iii) $|\mathbf{F_1}|$. (1 mark)

(b) The resultant of $3\mathbf{F_1}$, $6\mathbf{F_2}$ and a third force is $5\mathbf{i} + 4\mathbf{j}$ N. Determine the magnitude of the third force. (3 marks)

Question 2 **(6 marks)**

(a) Consider the statement: $n = 2 \Rightarrow n^2 = 4$.

(i) Write the inverse statement. (1 mark)

(ii) Write the converse statement. (1 mark)

(b) State whether each of the following statements are true or false, supporting each answer with an example or counterexample.

(i) $\forall$ positive integers $x, \sqrt{x} \leq x$. (2 marks)

(ii) $\forall a \in \mathbb{R}, \exists b \in \mathbb{R}$ such that $ab = 24$. (2 marks)

Question 3 **(4 marks)**

Prove:

$$\frac{\tan^2 x - 1}{\sin x + \cos x} = \frac{\sin x - \cos x}{\cos^2 x}$$

See next page

Question 4 **(6 marks)**

The position vectors of points A and B are $\mathbf{r}_A = \begin{pmatrix} -8 \\ 3 \end{pmatrix}$ and $\mathbf{r}_B = \begin{pmatrix} 7 \\ -2 \end{pmatrix}$.

(a) Determine the position vector of point P such that $\boldsymbol{AP} = \frac{2}{5}\boldsymbol{AB}$.

(3 marks)

(b) A small body leaves A and moves with a constant velocity in a direction parallel to $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Determine, with reasons, whether the body will pass through point C with position vector $\mathbf{r}_C = \begin{pmatrix} 6 \\ 9 \end{pmatrix}$. (3 marks)

Question 5 **(7 marks)**

(a) 4 different letters are chosen from the 7 in the word PAYMENT and then arranged to form a password. Determine how many different passwords are possible that

(i) end in T. (1 mark)

(ii) end in T or start with P. (3 marks)

(b) Determine the number of two letter permutations that can be made using letters from the word REPAYMENT. (3 marks)

Question 6 **(7 marks)**

Trapezium $OABC$ is such that $\overrightarrow{AB} = 3\overrightarrow{OC}$.

The midpoints of sides OA, AB, BC and OC are P, Q, R and S.

Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. Use a vector method to prove that $PQRS$ is a parallelogram.

Question 7 **(7 marks)**

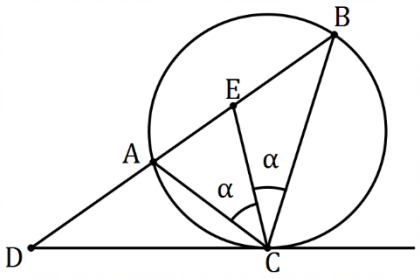
Consider the vectors $\mathbf{p} = \begin{pmatrix} -7 \\ 8 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

(a) Determine the vector projection of $\mathbf{r}$ onto $\mathbf{q}$. (3 marks)

(b) Given that $\mathbf{p} = \lambda\mathbf{q} + \mu\mathbf{r}$, determine the value of λ and the value of μ. (4 marks)

Question 8 **(7 marks)**

In the diagram shown, A, B and C lie on a circle. The tangent at C and secant BA intersect at D. Point E lies on AB so that CE bisects $\angle ACB$.



(a) Show that $\angle DEC = \angle DCE$. (3 marks)

(b) Given that $AE = 4$ cm and $BE = 9$ cm, determine the length of DC. (4 marks)

End of questions

Supplementary page

Question number: _____